I.E.S/I.S.S.
Examination - 2025

BGSP–F–STS

STATISTICS

PAPER—IV

Time Allowed : Three Hours	Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are **FOURTEEN** questions divided under **SEVEN** Sections.

Candidate has to choose any **TWO** Sections and attempt the questions therein. All the Sections carry equal marks. Candidate has to write answers at the specified space given for attempting First Choice Section and Second Choice Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Diagrams/Figures, wherever required, shall be drawn in the space provided for answering the question itself. Graph sheets, Normal Area Table and *F* Table can be found at the end of both choices of Sections in the Booklet.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.

SECTION—A

(Operations Research and Reliability)

1. Answer *all* of the following : 10×5=50

(a) Data scientists in the data analytics section of a multinational company are expected to increase to a strength of 30 and then remain at that level. The attrition of recruits depends on their length of service and it is as follows :

Year	*Total percent who left up to the end of year*
1	10
2	38
3	60
4	72
5	90
6	100

(i) Determine the number of data scientists to be recruited every year.

(ii) If there are four posts of team leaders for which length of service is the only criterion of promotion, what will be the average length of service after which a new entrant should expect promotion? 10

(b) A salesman wishes to visit cities 1, 2, 3 and 4. He does not want to visit any city twice before completing the tour of all the cities and wishes to return to his home city. The cost of going from one city to another, in rupees, is given in the table. Find the least cost route :

		To city			
		1	*2*	*3*	*4*
From city	*1*	0	15	40	25
	2	20	0	70	15
	3	20	25	0	10
	4	35	40	65	0

10

(c) When is a rectangular game said to be strictly determinable? For what values of x, the game with the following payoff matrix is strictly determinable?

$$\begin{array}{cc} & \begin{array}{ccc} \textit{Player B} & & \\ \textit{I} & \textit{II} & \textit{III} \end{array} \\ \begin{array}{c} \\ \textit{Player A} \\ \\ \end{array} & \begin{array}{c} \textit{I} \\ \textit{II} \\ \textit{III} \end{array} \begin{bmatrix} x & 5 & 1 \\ -2 & x & -8 \\ -3 & 3 & x \end{bmatrix} \end{array}$$

10

(d) The probability density function of time to failure for a system is

$$f(t) = \frac{t}{100} e^{\frac{-t^2}{200}}, \quad t > 0$$

(i) Find the reliability function.

(ii) Find the failure rate function and comment on the monotonicity of failure rate.

(iii) Find the mean time to failure. 10

(e) If the failure rate of a random variable T is $h(t) = abt^{b-1}$, $t > 0$, obtain the probability density function and reliability function of T. 10

2. Answer any *two* from the following : 25×2=50

(a) (i) Describe the discrete case of a stochastic inventory model when the probability distribution of demand is known.

(ii) A newspaper boy buys papers for seven rupees and sells them for ten rupees each. He cannot return unsold newspapers. The daily demand has the following probability distribution :

Number of customers	*Probability*
55	0·01
56	0·05
57	0·14
58	0·18
59	0·42
60	0·20

If each day's demand is independent of the previous day's demand, how many papers should he order each day? 25

(b) (i) For M|M|1 : (N|FIFO) queue system, derive the steady-state distribution of the queue length.

(ii) At a railway station, only one train is handled at a time. The railway yard has enough capacity for only two trains to wait while another is given the signal to depart. Trains arrive at the station at an average rate of 5 per hour and the railway station can handle them on an average of 10 per hour. Assuming Poisson arrivals and exponential service distribution, (1) compute the steady-state probabilities for the distinct number of trains in the system and (2) determine the average waiting time of a new train coming into the yard. 25

(c) The lifetime of a component has a distribution with the cumulative distribution function $F(t) = 1 - e^{-\theta t}$, $t \geq 0$, $\theta > 0$ and assume that immediately after the component fails, it is replaced by another component with exactly the same distribution of lifetime.

(i) What will be the distribution of the number of failures in a period of fixed length h? Justify your claim.

(ii) Determine the expected number of failures in the period of fixed length h. 25

(d) Solve the following LPP by dynamic programme approach :

Maximize $Z = 7x_1 + 8x_2$

subject to

$$x_1 + 2x_2 \leq 8$$

$$2x_1 + 5x_2 \leq 15$$

$$x_1, x_2 \geq 0$$

25

SECTION—B

(Demography and Vital Statistics)

3. Answer *all* of the following : 10×5=50

(a) Explain gross and net reproduction rates. Discuss their suitability as measures of population growth. 10

(b) Describe the characteristic equation of stable population, stating the underlying assumptions. 10

(c) Describe the indirect measures of net internal migration. 10

(d) Starting from a suitable assumption regarding relative growth rate of population, derive the logistic curve. 10

(e) Describe the different methods of computing infant mortality rates. 10

4. Answer any *two* from the following : 25×2=50

(a) Describe the different important sources of demographic data in India. 25

(b) Complete the following life table :

Age (years)	l_x	d_x	q_x	p_x	L_x	T_x	e_x°
92	4412	×	×	×	×	×	×
93	3724	×	×	×	×	×	×
94	3201	642	×	×	×	26567	×

25

(c) Describe the different measures of fertility with their relative merits and demerits. 25

(d) Find the probability that a person aged 30 will *(i)* survive to age 40, *(ii)* die before age 40 and *(iii)* die between ages 60 and 80 from the following extract of an abridged life table :

Age (x)	0	10	20	30	40	50	60	70	80	90	100	110
$l(x)$	1000	950	931	913	885	842	772	674	450	210	090	000

7+9+9=25

SECTION—C

(Survival Analysis and Clinical Trials)

5. Answer *all* of the following : 10×5=50

(a) Describe random censoring. Derive the likelihood function of a random sample subject to random censoring. 10

(b) Let $X_1, X_2, \ldots, X_n$ be the lifetimes of n individuals. Associated with each X_i, there is a censoring variable C_i. Clearly stating the conditions, derive the Kaplan-Meier estimator and state *(i)* how to adjust for ties and *(ii)* how to define the estimator when the last ordered lifetime is censored. 10

(c) Define IFR and DMRL classes. Give the implications of these classes. 10

(d) *(i)* Describe the competing risk setup and explain why the model is not fully identifiable.

(ii) Establish the relationship between overall hazard and cause-specific hazards, and hence obtain the marginal survival function. 5+5=10

(e) Define and distinguish among selection bias, information bias and confounding. How do these differ from random error and what are the strategies to minimize their impact? 10

6. Answer any *two* from the following : 25×2=50

(a) If $h(t)$ is the failure rate and $m(t)$ is the mean residual life function, it is observed that $h(t)\,m(t) = 1$, for all t. Derive the distribution function for the underlying random variable. 25

(b) Suppose that a population contains individuals for which T is exponentially distributed, but the hazard rate λ varies across the individuals and $f(t|\lambda) = \lambda e^{-\lambda t}$, $t \geq 0$ and λ itself has a gamma distribution. Find the unconditional distribution of T and its hazard function. 25

(c) Describe the Gehan's test. Consider two treatments A and B administered to five patients each. The survival time is given as follows :

A : 3, 5, 7, 9+, 18

B : 12, 19, 20, 20+, 33+

Compute the Gehan's statistic. Assuming that the variance of statistic is 79·44, test if the treatments have same effect. 25

(d) Compare and contrast parallel-group and crossover design in clinical trials, highlighting the advantages and disadvantages of each. 25

SECTION—D

(Quality Control)

7. Answer *all* of the following : 10×5=50

(a) List at least five criteria for detecting lack of control in $\bar{X}$ and R charts. 10

(b) Assume that the probability for a sample mean to fall above the central line is $\frac{1}{2}$. If there has been no shift in the population average, what is the probability that a random occurrence of *(i)* seven points to fall either above or below the central line, and *(ii)* at least ten out of eleven points falling on the same side of the central line? 5+5=10

(c) Seven inspectors, each inspected 1500 items for defects. The numbers of errors committed by them are given below :

Inspector Id	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Number of errors	4	6	6	2	15	4	4

Prepare a suitable chart to see whether all the inspectors' errors are within a reasonable control. If they are not in control, identify the inspector who is 'out of control'. 10

(d) Obtain the functions of OC curve for each of the following schemes :

(i) A single item is taken from a lot. If it is defective, the lot is rejected; otherwise it is accepted.

(ii) A sample of three items is selected from a lot. If the sample contains less than two defective items, the lot is accepted; otherwise it is rejected. 5+5=10

(e) Let p denote the probability that a sample point in the $\bar{X}$ chart to lie above UCL. If R denotes a random variable taking value r, ($r = 1, 2, \ldots$), so that r th sample point is the first one to go above UCL, then show that $E[R] = \frac{1}{p}$. 10

8. Answer any *two* from the following : 25×2=50

(a) A machine is automatically filling up a large number of cans with prefixed quantity of 40 mL of a juice. A sequence of random collection of ten samples each containing four cans is selected and measured its filled up quantity. The details are given below :

Quantity filled up (in mL)

Sample No.	*Can-1*	*Can-2*	*Can-3*	*Can-4*
1	41	40	42	39
2	42	43	42	44
3	41	42	42	41
4	38	39	42	39
5	43	39	39	44
6	41	39	39	40
7	42	43	37	42
8	44	42	41	43
9	43	42	42	44
10	40	42	42	43

(i) Plot the data and draw suitable control lines. Comment on the process control.

(ii) If the specification limits are 40 ± 4 mL, is the process capable of meeting it?

(iii) Calculate the fraction defective of the production process, if the defective is the one having larger filled up can.

(Given : $A_2 = 0 \cdot 729$, $D_4 = 2 \cdot 282$, $D_3 = 0$, $d_2 = 2 \cdot 059$, for $n = 4$) 15+5+5=25

(b) Show that the probability that at least two points, one in $\bar{X}$ chart and one in R chart, go outside of control limits is

$$1 - [\phi(\sqrt{n}\,T + 3\theta) - \phi(\sqrt{n}\,T - 3\theta)] \cdot \left[\Pr\left(\frac{R}{\sigma} \le D_2\theta\right) - \Pr\left(\frac{R}{\sigma} \le D_1\theta\right) \right]$$

where $\theta = \dfrac{\sigma'}{\sigma}$, $T = (\bar{X}' - \bar{X})/\sigma$, $\phi(x) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du$, and D_1 and D_2 are constants used to estimate σ, assuming that the control charts are based on assumed $N(\bar{X}', \sigma')$, where the actual population is $N(\bar{X}, \sigma)$. 25

(c) A certain manufacturing process produces items whose measured characteristics are known to have trend over time. The sample averages and ranges of fifteen subgroups, of five items, are given below :

($\bar{X}$, R) values are

(198, 7), (197, 2), (204, 10), (203, 12), (205, 17), (204, 9), (205, 10), (210, 9), (209, 7), (207, 16), (210, 9), (214, 8), (211, 16), (211, 6), (213, 8)

(i) Construct a trended $\bar{X}$ control chart and R chart.

(ii) Comment whether the process is under control.

(Given : $A_2 = 0 \cdot 58$, $D_4 = 2 \cdot 114$, for $n = 5$) 25

(d) Explain the following :

(i) Cumulative sum chart

(ii) EWMA chart

(iii) AOQ and AOQL

(iv) ATI for single and double sampling plans

(v) LTPD 5+5+5+5+5=25

SECTION—E

(Multivariate Analysis)

9. Answer *all* of the following : 10×5=50

(a) Let $X = (X_1 \ X_2 \ X_3)' \sim N_3(0, \Sigma)$, where

$$\Sigma = \begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & -\rho \\ 0 & -\rho & 1 \end{pmatrix}$$

Determine the value of ρ for which $X_1 + X_2 + X_3$ and $X_1 - X_2 - X_3$ are statistically independent. 10

(b) Let X_1, X_2, X_3, X_4 be independent and identically distributed normal $N(0, \sigma^2)$ random variables. Obtain the distributions of (i) $X_1X_2 - X_3X_4$ and (ii) $X_1X_2 + X_3X_4$. 10

(c) If $X = (X_1 \ X_2 \ X_3)' \sim N_3(\mu, \Sigma)$ with

$$\mu = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

find the expected value of $X_1^2 + X_2^2 + 2X_1X_3 + 2X_2X_3$. 10

(d) If the Wishart matrix is partitioned as

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

obtain the distribution of $V_{11\cdot 2} = V_{11} - V_{12}\, V_{22}^{-1}\, V_{21}$. 10

(e) What are canonical variables and canonical correlations? Briefly explain how one can obtain the first pair of canonical variables and the first canonical correlation. 10

10. Answer any *two* from the following : 25×2=50

(a) (i) Let $X = (X_1\ X_2)' \sim N_2(\mu, \Sigma)$ and let $Y_1 = X_1 - X_2$ and $Y_2 = X_1 + X_2$. If $Y = (Y_1\ Y_2)' \sim N_2(\mu_*, \Sigma_*)$ with $\mu_* = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\Sigma_* = \begin{pmatrix} 3 & -1 \\ -1 & 7 \end{pmatrix}$, obtain the values of μ and Σ.

(ii) Briefly explain how the principal component analysis is useful for dimension reduction. Show that the principal components are uncorrelated. 25

(b) Let $Z = (X_1\ X_2\ X_3\ X_4)' \sim N_4(\mu, \Sigma)$, in which

$$\mu = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 1 & 2 & 1 & -1 \\ 2 & 2 & -1 & 1 \\ 1 & -1 & 1 & -2 \\ -1 & 1 & -2 & 1 \end{pmatrix}$$

Determine the conditional distribution of $(X_1\ X_2)'$ given $(X_3\ X_4)'$, and write down the corresponding mean vector and dispersion matrix. 25

(c) Let $V = \begin{pmatrix} X & Y \\ Y & Z \end{pmatrix}$ follow Wishart distribution $W_2(n, I)$. Obtain the distributions of

(i) $U_1 = \frac{1}{4}X - \frac{1}{3}Y + \frac{1}{9}Z$ and (ii) $U_2 = \begin{pmatrix} X - 4Y + 4Z & -3X + 7Y - 2Z \\ -3X + 7Y - 2Z & 9X - 6Y + Z \end{pmatrix}$. 25

(d) (i) Explain Fisher's discriminant function. How would you test whether the constructed discriminant function can discriminate two multivariate normal populations?

(ii) Describe a test based on Mahalanobis D^2 statistic for testing the equality of means of two multivariate normal populations with common dispersion matrix. 25

SECTION—F

(Design and Analysis of Experiments)

11. Answer *all* of the following : 10×5=50

(a) Stating the one-way ANOVA model, describe how to check graphically the assumptions associated with it. 10

(b) Describe the basic principles of experimental design and explain how it is implemented in the Latin square design. 10

(c) How is partial confounding different from total confounding? Illustrate using a 2^3 factorial design replicated four times giving the partition of degrees of freedom. 10

(d) In a two-factor experimental design with only a single replicate, the effects model is given under standard notations as

$$y_{ij} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ij}, \quad i = 1, 2, \ldots, a; \quad j = 1, 2, \ldots, b$$

and $\varepsilon \sim N(0, \sigma^2)$.

(i) Under this model, explain why σ^2 is unestimable and tests for main effects cannot be constructed.

(ii) Modify the model appropriately and propose an estimate for σ^2 along with tests for the main effects. 5+5=10

(e) A dietician is studying how exercise intensity and diet type affect weight loss, and has carried an experiment with layout of sample size as follows :

Exercise intensity / *Diet*	*High*	*Low*
Diet I	2	8
Diet II	5	15

Why is this a non-orthogonal design? What are its statistical implications? 10

12. Answer any *two* from the following : 25×2=50

(a) A block officer wishes to test the effect of five different irrigation techniques on yield of certain crop. He implemented the five different techniques on four fields and observed the yield in tons. The design layout is as follows :

	Irrigation techniques				
Fields	I_1	I_2	I_3	I_4	I_5
F_1	20	15	35	20	15
F_2	30	20	30	20	10
F_3	15	30	50	35	40
F_4	40	40	20	40	30

Examine if analysing this data as RBD is more effective than analysing this as CRD. 25

(b) How many treatment combinations are there in a 3^3 factorial experiment? Assuming that only nine experimental runs are feasible and blocking is possible, a Latin square design is incorporated to this. Give the layout and analyse the distribution of the degrees of freedom. Comment and discuss if all factorial effects can be estimated. If not, give the effects that cannot be estimated. 25

(c) A study on salaries of faculties in Science, Humanities and Technology was carried out and the data is presented below. Using $\alpha = 0 \cdot 05$ level of significance, test if there is any difference in salaries among faculties in the three mentioned areas :

Science		*Humanities*		*Technology*	
Salary (in thousands)	*Year of experience*	*Salary* (in thousands)	*Year of experience*	*Salary* (in thousands)	*Year of experience*
35	2	68	28	46	5
47	7	54	17	39	1
65	22	38	6	47	7
51	14	59	19	63	18
45	4	47	10	63	22

25

(d) A manufacturing company is conducting an experiment to study the effects of curing temperature—high and low; pressure at moulding—low, medium and high. The experiment is conducted in a setting where it is easier to control the

curing temperature for entire batches but more practical to apply pressures to smaller sections within each batch.

(i) Explain why a split-plot design is appropriate here. Identify the factors to be assigned to plots and give the linear model for split-plot design considering the interactions too.

(ii) Suppose one is to use a strip-plot design instead, how will the treatments be arranged?

(iii) It is suspected that the factors interact. In that case, which of the above designs will you recommend and why? 10+10+5=25

SECTION—G

(Computing with C and R)

13. Answer *all* of the following : 10×5=50

(a) What are 'bitwise operators' and 'bitwise shift operators'? Give examples. 10

(b) Write a C program to check whether a given year is a leap year or not. 10

(c) Write a C program to calculate the sum of the expression

$$\frac{1}{2}+\frac{5}{6}+\cdots+\frac{99}{100}$$

10

(d) Explain any five file operations in C language. 10

(e) Write a set of R codes to *(i)* read a given vector of values, *(ii)* create a matrix of dimensions $r_1 \times c_1$, *(iii)* compute minmax and maxmin values and *(iv)* make a printout on whether these values are equal or not. 10

14. Answer any *two* from the following : 25×2=50

(a) What is a linked list? Explain different types of linked lists. State the basic operations that can be carried on them. 25

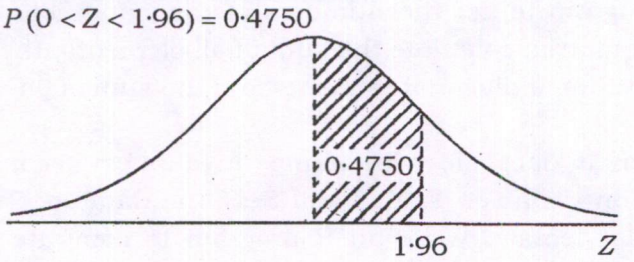
(b) *(i)* What are the differences between local and global variables?

(ii) What are structures and union? Explain the differences between them. 10+15=25

(c) Write a C program to get the numbers of rows and columns, and individual elements of a matrix; calculate the sum of all elements, the sum of elements in each row, the sum of diagonal elements and the sum of off-diagonal elements. 25

(d) You are given a data file with name "mydata.csv", containing 25 rows of observations on variables Y, X, Z and Sex. The data on 'Sex' is coded as 1 for male and 0 for female. Write an R program to compute multiple regression coefficients of Y on X, Z separately for males and females, without using the built-in function "lm( )". 25

★ ★ ★



The Normal Distribution

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Percentage Points of the *F* Distribution

$F_{0.05,\, \nu_1,\, \nu_2}$

ν_2 \ ν_1	Degrees of Freedom for the Numerator (ν_1)																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Degrees of Freedom for the Denominator (ν_2)