I.E.S/I.S.S.
Examination - 2025

BGSP–T–STS

STATISTICS

PAPER—III

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are **EIGHT** questions divided under **TWO** Sections.

Candidate has to attempt **FIVE** questions in all.

Both the questions in Section—A are **compulsory**.

Out of the **SIX** questions in Section—B, any **THREE** questions are to be attempted.

The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer (QCA) Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.

SECTION—A

1. *(a)* If the cost function is of the form $C = C_0 + \Sigma t_i \sqrt{n_i}$, where C_0 and t_i are known numbers, then in order to minimize $V(\bar{y}_{st})$ for fixed total cost, show that n_i must be proportional to

$$\left(\frac{W_i^2 \, S_i^2}{t_i} \right)^{2/3}$$

15

(b) Define the general form of the simultaneous equations model. Based on the likelihood function, explain the problem of identification. When is an equation called just identified, under-identified or over-identified? Also, give the order condition for identification. 15

(c) Show that the demand curve with constant price elasticity of demand is a hyperbola whose shape depends on the value of the parameter. Hence, show that when the elasticity of demand is 1, then the demand curve is an equilateral hyperbola. 10

2. *(a)* Consider the following observations of a population of size 5 :

110, 60, 80, 20, 30

If a simple random sample of size 2 is taken from this population, then using the given observations, show that simple random sampling without replacement gives a more efficient estimator of population mean than simple random sampling with replacement. 15

(b) In general linear model $y = X\beta + u$, how does the presence of multicollinearity affect the squared length of the OLS estimator of β? Describe multicollinearity index and variance inflation factor for the detection of multicollinearity. 10

(c) Define the following terms :

(i) Autocovariance function

(ii) Autocorrelation function

(iii) Spectral density function

For the stochastic processes

$$Z_t = 10 + a_t + 0 \cdot 7 a_{t-1}$$

$$Z_t = 10 + a_t - 0 \cdot 7 a_{t-1}$$

where $\{a_t\}$ is a sequence of uncorrelated $N(0, 1)$ variables, obtain the covariance function and spectral density function. 15

SECTION—B

3. *(a)* Define product estimator for estimating population total. Obtain its bias and variance to the first order of approximation. 15

(b) Derive the mixed regression estimator for the regression coefficients vector of a general linear model involving stochastic linear restrictions for the regression parameters. Show that the resulting estimator is unbiased. Obtain the variance-covariance matrix of the mixed regression estimator and prove that the difference between the variance-covariance matrix of OLS estimator and that of mixed regression estimator is positive definite. 15

(c) Define time series. When is a time series called stationary time series? Give an example of stationary time series. Check the stationarity of the following time series y_t :

$$y_t = y_{t-1} + u_t$$ 10

4. *(a)* A village has 10 orchards containing 150, 80, 50, 200, 100, 40, 160, 60, 140 and 220 trees respectively. Select a sample of 3 orchards with probability proportional to the number of trees in the orchards by cumulative total procedure if the random numbers drawn from the random number table are 850, 300 and 650. 10

(b) Consider the general structural model (the symbols have their usual meanings)

$$\underset{(n\times M)}{y}\underset{(M\times M)}{B'} + \underset{(n\times K)}{X}\underset{(K\times M)}{\Gamma'} = \underset{(n\times M)}{U}$$

For estimating the coefficients of the first equation

$$\underset{(n\times 1)}{y_1} = \underset{(n\times m-1)}{Y_1}\underset{(m-1\times 1)}{\beta} + \underset{(n\times k)}{X_1}\underset{(k\times 1)}{\gamma} + \underset{(n\times 1)}{u_1}$$

which is assumed to be (just or over) identified, derive the two-stage least squares (2SLS) estimator. Prove that for the just identified case, the 2SLS estimator reduces to the indirect least squares (ILS) estimator. 15

(c) Explain briefly the major steps involved in the applications of the Box-Jenkins methodology for forecasting. Interpret the following model of time series y_t for forecasting :

$$\hat{y}_t^* = 23\cdot 08 + 0\cdot 34 y_{t-1}^* - 0\cdot 29 y_{t-8}^* - 0\cdot 26 y_{t-12}^*$$

where $y_t^* = (y_t - y_{t-1})$ 15

5. (a) Describe non-sampling errors and their sources. 15

(b) Consider the linear regression model $y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + u_i$; $i = 1, \cdots, n$ with

$$\sum_i x_{1i} = \sum_i x_{2i} = 0, \quad \sum_i x_{1i}^2 = \sum_i x_{2i}^2 = 1, \quad \sum_i x_{1i} x_{2i} = \rho$$

Obtain the OLS estimator of $\beta = (\beta_1 \ \beta_2)'$. Derive the variance-covariance matrix and explain how the presence of multicollinearity affects the performance of the OLS estimators of β_1 and β_2. 10

(c) Describe briefly the various methods of weighted indices for constructing an index number of prices.

A worker in the city of Ahmedabad earns ₹ 45,000 per month. The cost of living index for a particular month is given as 130. Using the following information, find the amount of money he spent on clothing and house rent :

Group	*Expenditure* (in ₹)	*Group index*
Food	14,000	180
Clothing	?	150
House rent	?	100
Fuel and lighting	6,600	110
Miscellaneous	7,500	80

15

6. (a) Define Horvitz-Thompson estimator for estimating population mean. Show that it is unbiased and give its variance. What is the main drawback of this estimator and how can it be tackled? Explain. 15

(b) Consider the following structural form of the simultaneous equations model :

$$y_{1t} + \beta_{12} y_{2t} + \gamma_{11} x_{1t} + \gamma_{12} x_{2t} = u_{1t}$$

$$y_{2t} + \beta_{21} y_{1t} + \gamma_{23} x_{3t} = u_{2t}$$

Here *y*'s are the endogenous and *x*'s are the predetermined variables.

(i) Obtain the reduced form of the model.

(ii) Using rank and order conditions, check the identifiability of both the equations of the model. 15

(c) Describe briefly the various mathematical tests of consistency for the construction of index numbers. Show that Fisher's index number does not satisfy the circular test using proper example. 10

7. (a) A sampler proposes to draw a stratified random sample. He expects that his field cost will be of the form $\sum C_i n_i$. His advance estimates of relevant quantities for the two strata are as follows :

Strata	*Size of strata*	C_i (in ₹)	S_i
1	400	4	10
2	600	9	20

(i) Find the values of n_1 and n_2 that minimize the variance of the estimator of population mean if the sample size is 264.

(ii) What will be the total field cost? 10

(b) Consider the multiple linear regression model

$$y = X\beta + u; \quad E(u) = 0, \quad E(uu') = \sigma^2 I_n$$

(i) Derive an ordinary least squares (OLS) estimator of β and its variance-covariance matrix.

(ii) For a forecast period, say f, with X variables uncertain, give unbiased point prediction and obtain its variance. 15

(c) If $U = CX^{\alpha} Y^{\beta}$ is an individual's utility function of two commodities, show that his demand for the goods is

$$x = \frac{\alpha}{\alpha+\beta} \cdot \frac{m}{p_x}, \quad y = \frac{\beta}{\alpha+\beta} \cdot \frac{m}{p_y}$$

where p_x, p_y are the fixed prices of the two commodities, and m is the individual's fixed income. Deduce that the elasticity of demand for either good with respect to income or price is equal to unity in absolute value. 15

8. (a) Explain double sampling. Define double sampling ratio estimator for estimating the population mean and obtain an expression for its variance to the first order of approximation. 15

(b) For the linear model with AR(1) disturbances $y_t = \beta x_t + u_t;\ u_t = \rho u_{t-1} + \varepsilon_t$, obtain the OLS estimator of β and derive its variance. Compare it with the variance of OLS estimator when no autocorrelation is present. Comment on the effect of ignoring autocorrelation. 10

(c) Define demand and supply functions and price elasticities of demand and supply functions. Explain how the change in price on the total expenditure made by the population affects the purchase of commodity using price elasticity of demand. 15

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