

**DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO**

**T.B.C. : BGSP-B-ST**

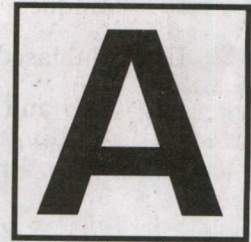


**Test Booklet Series**

1006757



**TEST BOOKLET  
STATISTICS**



**Paper II**

**Time Allowed : Two Hours**

**Maximum Marks : 200**

**INSTRUCTIONS**

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET **DOES NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside.  
**DO NOT** write *anything else* on the Test Booklet.
4. This Test Booklet contains **80** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.
5. You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. **All** items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator **only the Answer Sheet**. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**  
**THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.**
  - (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
  - (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
  - (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

**DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO**

1. Consider the following statements :

- I. MLEs are unbiased but not necessarily unique.
- II. Unbiased estimator is always unique.
- III. If  $U$  and  $W$  are consistent estimators of  $\theta_1$  and  $\theta_2$ , then  $UW$  is also consistent for  $\theta_1\theta_2$ .

Which of the statements given above is/are correct ?

- (a) I and II
- (b) I and III
- (c) II and III
- (d) III only

2. With the increase in sample size if the estimator becomes closer and closer to the parameter, then it is called :

- (a) Completeness
- (b) Consistent
- (c) Sufficient
- (d) Unbiasedness

3. Let  $X_1, X_2, X_3, \dots, X_n \sim N(\mu, \sigma^2)$  where mean  $\mu$  and variance  $\sigma^2$  are unknown. Let  $S^2 = \frac{1}{n} \sum (X_i - \bar{X})^2$  and  $s^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$ .

Which one of the following is correct according to the efficiency criterion ?

- (a)  $s^2$  is more efficient than  $S^2$ .
- (b)  $s^2$  is less efficient than  $S^2$ .
- (c)  $s^2$  and  $S^2$  cannot be compared.
- (d) Efficiency of  $s^2$  is equal to efficiency of  $S^2$ .

4. Suppose  $X_1, X_2, X_3, \dots, X_n$  be a random sample of size  $n$  from Poisson distribution with parameter  $\lambda$ . Then uniformly minimum variance unbiased estimator (UMVUE) of  $\lambda$  is :

- (a)  $\frac{\sum_{i=1}^n X_i^2}{n}$
- (b)  $\frac{\sum_{i=1}^n X_i}{n+1}$
- (c)  $\frac{\sum_{i=1}^n X_i}{n}$
- (d)  $\frac{\sum_{i=1}^n X_i^2}{n+1}$

5. Let  $S$  be the set of all unbiased estimators  $T$  of  $\theta \in \Theta$  such that  $E_\theta T^2 < \infty$  for all  $\theta \in \Theta$ . An estimator  $T_0 \in S$  is called a uniformly minimum variance unbiased estimator (UMVUE) of  $\theta$  for  $T \in S$  if :

- (a)  $E_\theta(T_0 - \theta) \leq E_\theta(T - \theta)$
- (b)  $E_\theta(T_0 - \theta)^2 > E_\theta(T - \theta)^2$
- (c)  $E_\theta(T_0 - \theta)^2 \leq E_\theta(T - \theta)^2$
- (d)  $E_\theta(T_0 - \theta) > E_\theta(T - \theta)$

6. Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from  $N(\mu, \sigma^2)$ . Consider the statistic  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ . What is  $V(s^2)$  equal to ?

- (a)  $\frac{2\sigma^4}{n-1}$
- (b)  $\frac{2\sigma^2}{n-2}$
- (c)  $\frac{2\sigma^2}{n-1}$
- (d)  $\frac{2\sigma}{n-1}$

7. Let  $X_i \sim N(\mu, \sigma^2)$ ;  $i = 1, 2, 3, \dots, n$ . Which of the following statements is/are correct ?

- I. At confidence level  $(1 - \alpha)$  if  $\alpha$  and  $\sigma$  are known, then we can choose sample size  $n$  to get a confidence interval of a fixed length.
- II. At confidence level  $(1 - \alpha)$  if  $\sigma$  is not known, then we cannot choose sample size  $n$  to get a fixed width confidence interval of level  $(1 - \alpha)$ .

Select the answer using the code given below :

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

8. Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample of size  $n$  taken from  $U(\theta - 0.5, \theta + 0.5)$  where  $\theta \in (-\infty, \infty)$ . If  $T_1$  and  $T_2$  are respectively the minimum and maximum order statistics, then which of the following statements is/are correct regarding the statistic  $T = T_2 - T_1$  ?

- I.  $T$  is sufficient statistic of  $\theta$ .
- II.  $T$  has a complete family of distributions.

Select the answer using the code given below :

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

9. Let  $X_1, X_2, X_3, \dots, X_n, X_{n+1}$  be  $(n+1)$  observations from  $N(\mu, 1)$  distribution. If  $\bar{X}_n = \frac{\sum_{i=1}^n X_i}{n}$  and  $T = \frac{(\bar{X}_n + X_{n+1})}{2}$ , then which one of the following is correct ?

- (a)  $T$  is an unbiased but inconsistent estimator of  $\mu$ .
- (b)  $T$  is a biased but consistent estimator of  $\mu$ .
- (c)  $T$  is an unbiased and consistent estimator of  $\mu$ .
- (d)  $T$  is neither an unbiased nor a consistent estimator of  $\mu$ .

10. In sampling from a location-scale family,  $f(x; \theta, \sigma) = \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right)$ , which one of the following is a pivot ?

- (a)  $(\bar{X} - \theta)$
- (b)  $(X_{(2)} + X_{(1)} - 2\theta)$
- (c)  $\frac{(\bar{X} - 2\theta)}{\left(\frac{s}{\sqrt{n}}\right)}$
- (d)  $\frac{X_{(2)} + X_{(1)} - 2\theta}{s}$

11. If  $C$  is the critical region and  $H_0$  is a composite hypothesis, then the level of significance  $\alpha$  is :

(a)  $\alpha = \sup_{h \in H_0} P(x \in C | h)$

(b)  $\alpha = \sup_{h \in H_0} P(x \in C^c | h)$

(c)  $\alpha = \inf_{h \in H_0} P(x \in C | h)$

(d)  $\alpha = \sup_{h \in H_1} P(x \in C^c | h)$

12. For testing  $H_0 : f(x) = \begin{cases} \frac{x}{2}; & 0 < x < 2 \\ 0; & \text{otherwise} \end{cases}$  against

$H_1 : f(x) = \begin{cases} \frac{3x^2}{8}; & 0 < x < 2 \\ 0; & \text{otherwise} \end{cases}$  based on a single

observation, the size of the test was chosen to be  $\alpha = 0.36$ .

Which one of the following statements is/are correct ?

I. The power of most powerful test of given size is 0.488.

II. The specified test is unbiased.

Select the answer using the code given below :

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

13. For testing  $H_0 : f(x) = \begin{cases} 2x; & 0 < x < 1 \\ 0; & \text{otherwise} \end{cases}$  against

$H_1 : f(x) = \begin{cases} 3x^2; & 0 < x < 1 \\ 0; & \text{otherwise} \end{cases}$  on the basis of a

single observation, the power of a most powerful test of size  $\alpha = 0.19$  is :

(a) 0.271

(b) 0.275

(c) 0.729

(d) 0.810

14. What is the distribution having MLR property required for UMP test ?

(a) Hypergeometric distribution

(b) Poisson distribution

(c) Cauchy distribution

(d) Normal distribution

15. Let (1.25, 0.85, 1.35) be a random sample from  $U(0, 2\theta)$ ,  $\theta > 0$ . What is the value of moment estimate of  $\theta$  ?

(a) 0.77

(b) 1.25

(c) 2.30

(d) 3.45

16. Consider a sequential probability ratio test for testing  $H_0 : \mu = \mu_0$  against  $H_1 : \mu = \mu_1$ , where  $\mu$  is the mean of normal distribution with variance unity. For this test, which of the following statements is/are correct ?

- I. The OC function  $L(\mu)$  may be obtained for a given  $\mu$  using the relation 
$$h(\mu) = \frac{\mu_1 - \mu_0 - 2\mu}{\mu_1 + \mu_0} \text{ in } L(\mu).$$
- II. If probabilities of errors  $\alpha$  and  $\beta$  involved in the test procedure are equal, then the average sample numbers under  $H_0$  and  $H_1$  are equal.

Select the answer using the code given below :

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

17. The largest order statistic  $X_{(n)}$  is complete and sufficient for which of the following ?

- I.  $X \sim U(0, \theta), \theta \in (0, \infty)$
- II.  $X \sim f(x) = \begin{cases} \frac{1}{N}; & x = 1, 2, 3, \dots, N \text{ where } N \geq 1 \\ 0; & \text{otherwise} \end{cases}$

Select the answer using the code given below :

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

18. Let  $X_i \sim N(\theta, \theta^2)$ . Then for parameter  $\theta, T = \left( \sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2 \right)$  is :

- (a) Sufficient
- (b) Complete
- (c) Sufficient and complete
- (d) Neither sufficient nor complete

19. In case of random sampling from  $F_\theta(X_i) \sim U(0, 1)$ , a pivot is :

- (a)  $\sum_{i=1}^n F_\theta(X_i)$
- (b)  $\sum_{i=1}^n \log F_\theta(X_i)$
- (c)  $\exp \{-F_\theta(X_i)\}$
- (d)  $-\sum_{i=1}^n \log F_\theta(X_i)$

20. Let  $X_i \sim U(0, \theta), \theta \in (0, \infty)$ . Then which of the following statements are correct ?

- I.  $X_{(n)}$  is maximum likelihood estimate of  $\theta$ .
- II.  $X_{(n)}$  is sufficient for  $\theta$ .
- III. Family of pdfs of  $X$  is complete.

Select the answer using the code given below :

- (a) I and II only
- (b) II and III only
- (c) I and III only
- (d) I, II and III

21. Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from pdf  $f(x) = \begin{cases} 1, & \theta \leq x \leq \theta + 1 \\ 0, & \text{otherwise} \end{cases}$ .

Which one of the following is correct ?

- (a) The smallest sample observation is a consistent estimator of  $\theta$ .
- (b) The largest sample observation is a consistent estimator of  $\theta$ .
- (c) Sample mean is a consistent estimator of  $\theta$ .
- (d) None of the above

22. Consider the following statements :

- I. Sample mean is an unbiased estimator of parameter of a Poisson distribution.
- II. Sample mean is a consistent estimator of population mean of a normal distribution.

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

23. Let  $X_1, X_2, X_3, \dots, X_n$  be iid random variables such that  $X_1 \sim N(\mu, \sigma^2)$ . The width of  $(1 - \alpha)$  level confidence interval  $(\bar{X} - c_1, \bar{X} + c_2)$  of  $\mu$  is :

- (a)  $\frac{2c\sqrt{n}}{\sigma}$  if  $c_1 = c_2 = c$
- (b)  $\frac{2c\sigma}{\sqrt{n}}$  if  $c_1 = c_2 = c$
- (c)  $\frac{(c_1 + c_2)\sigma}{2\sqrt{n}}$
- (d)  $\frac{\sqrt{2}(c_2 - c_1)\sigma}{\sqrt{n}}$

24. Let  $X \sim f(x; \theta)$  such that  $f(x; \theta) = \frac{2(x - \theta)}{(1 - \theta)^2}$ ,  $\theta < x < 1$ ,  $0 < \theta < 1$ . An unbiased estimator of  $\theta$  based on a sample of size one is given by :

- (a)  $\frac{2}{3}(1 - X)$
- (b)  $3(1 - 2X)$
- (c)  $(3X - 2)$
- (d)  $2(X - 3)$

25. Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from  $f(x) = \begin{cases} e^{-(x-\theta)}; & x > \theta \\ 0; & \text{otherwise} \end{cases}$ . The

unbiased estimator of  $\theta$  is :

- (a)  $\bar{X}$
- (b)  $\sum_{i=1}^n X_i - n$
- (c)  $\sum_{i=1}^n X_i - 1$
- (d)  $\bar{X} - 1$

26. Let  $x_1, x_2, x_3, \dots, x_n$  be a random sample from Bernoulli population with parameter  $p$ .

$T$  is defined as  $T = \begin{cases} \frac{1}{3n}, & \text{if } x_1 + x_2 + x_3 = 1 \\ 0, & \text{otherwise} \end{cases}$ .

Then  $T$  is unbiased estimator of :

- (a)  $\frac{p(1-p)}{n}$
- (b)  $\frac{p^2}{n}$
- (c)  $\frac{(1-p)^2}{n}$
- (d)  $\frac{p(1-p)^2}{n}$

27. Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from  $U[0, \theta]$  distribution. If  $T = X_{(n)} = \max(X_i)$ , then MVUE for  $\theta$  is :

- (a)  $\frac{(n+2)T}{n}$
- (b)  $\frac{(n+2)T}{n+1}$
- (c)  $\frac{(n+1)(n+2)T}{n}$
- (d)  $\frac{(n+1)T}{n}$

28. Consider the following statements :

- I. If  $X_1 \sim U(0, \theta)$ ,  $\theta \in (0, \infty)$ , then  $\max(X_1, X_2, X_3, \dots, X_n)$  is sufficient for  $\theta$ .
- II. If  $X_1 \sim f_\theta(x) = \begin{cases} 1, & \theta - 0.5 < x < \theta + 0.5 \\ 0, & \text{otherwise} \end{cases}$ , then  $(\min X_1, \max X_1)$  is jointly sufficient for parameter  $\theta$ .

Which of the statements given above is/are correct ?

- (a) I only  
(b) II only  
(c) Both I and II  
(d) Neither I nor II
29. A box contains  $N$  unknown items marked 1 through  $N$  where  $N \geq 2$ . The unbiased estimate of  $N$  after making  $n$  random selections from the box and observing  $X_1, X_2, X_3, \dots, X_n$  is :

- (a)  $\frac{\bar{X} + 1}{2}$   
(b)  $\frac{\bar{X} - 1}{2}$   
(c)  $2\bar{X} + 1$   
(d)  $2\bar{X} - 1$

30. Let  $x_1 = 3, x_2 = 4, x_3 = 3.5, x_4 = 2.5$  be four

observations from a distribution with pdf

$$f(x|\theta) = \frac{1}{3} \left[ \frac{1}{\theta} e^{-\frac{x}{\theta}} + \frac{1}{\theta^2} e^{-\frac{x}{\theta^2}} + e^{-x} \right]; \theta > 0. \text{ The}$$

method of moment estimator of  $\theta$  based on

above observations is :

- (a) 1  
(b) 2.5  
(c) 3.25  
(d) 3.5

31. Which one of the following provides lower bound to the variance of unbiased estimator ?

- (a) Factorization theorem  
(b) Rao-Blackwell theorem  
(c) Neyman-Pearson fundamental lemma  
(d) Cramer-Rao inequality

32. Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from a distribution with pdf  $f(x, \theta) = \theta x^{\theta-1}$ ;  $0 < x < 1, \theta > 0$ . Then the sufficient statistic for  $\theta$  is :

- (a)  $\sum_{i=1}^n X_i$   
(b)  $\sum_{i=1}^n X_i + 1$   
(c)  $\sum_{i=1}^n X_i^2$   
(d)  $\prod_{i=1}^n X_i$

33. Let  $X_1, X_2$  be a random sample of size 2 from Bernoulli distribution with pmf as  $f(x, \theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0, 1 \\ 0, & \text{otherwise} \end{cases}$ . Then  $X_1 + X_2$  is sufficient for  $\theta$  if  $P[X_1 = x_1, X_2 = x_2 | (X_1 + X_2) = t]$  is equal to :

- (a)  $\frac{1}{\binom{t}{2}}$   
(b)  $\frac{1}{\binom{2}{t}}$   
(c)  $\frac{1}{\binom{t+1}{2}}$   
(d)  $\frac{1}{\binom{2}{t+1}}$

34. The 95% confidence interval for  $\mu$  in  $N(\mu, \sigma^2)$  when  $\sigma$  is known is (200, 220). The value of sample mean is :

- (a) 420
- (b) 220
- (c) 210
- (d) 200

35. In a paired t-test, the observations are recorded in pairs. If the number of pairs is 20, then for testing  $H_0 : \mu_d = 0$  against  $H_1 : \mu_d \neq 0$ , the degrees of freedom for t-test are equal to :

- (a) 19
- (b) 18
- (c) 10
- (d) 9

36. Consider the problem of testing  $H_0 : \theta = \theta_0$  against  $H_1 : \theta = \theta_1$ . The critical region  $\omega$  and, consequently, test based on it, is said to be unbiased if :

- (a)  $1 - P(\text{Type-I error}) = P(\text{Type-II error})$
- (b)  $P(\text{Type-I error}) > P(\text{Type-II error})$
- (c)  $1 - P(\text{Type-II error}) > P(\text{Type-I error})$
- (d)  $P(\text{Type-I error}) < P(\text{Type-II error})$

**Consider the following for the next two (02) items :**

Let the probability density function

$$f_\theta(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, 0 < x < \infty.$$

37. What is the maximum likelihood estimator of  $\theta$  ?

- (a)  $\frac{1}{2} \bar{x}$
- (b)  $\bar{x}$
- (c)  $2\bar{x}$
- (d)  $4\bar{x}$

38. Consider the following statements :

- I. The estimator of  $\theta$  with methods of moment is equal to  $2\bar{x}$ .
- II. Both maximum likelihood estimator and method of moments estimator will be same.

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

**Consider the following for the next two (02) items :**

Let  $X_1, X_2, X_3, \dots, X_n$  be a sample from

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

39. What is the estimator of 'a' obtained by using method of moments ?

- (a)  $\bar{X} + \sqrt{\frac{4 \sum (X_i - \bar{X})^2}{n}}$
- (b)  $\bar{X} - \sqrt{\frac{3 \sum (X_i - \bar{X})^2}{n}}$
- (c)  $\bar{X} - \sqrt{\frac{2 \sum (X_i - \bar{X})^2}{n}}$
- (d)  $\bar{X} + \sqrt{\frac{\sum (X_i - \bar{X})^2}{n}}$

40. What is the estimator of 'b' obtained by using method of moments ?

- (a)  $\bar{X} - \sqrt{\frac{4 \sum (X_i - \bar{X})^2}{n}}$
- (b)  $\bar{X} + \sqrt{\frac{3 \sum (X_i - \bar{X})^2}{n}}$
- (c)  $\bar{X} + \sqrt{\frac{2 \sum (X_i - \bar{X})^2}{n}}$
- (d)  $\bar{X} - \sqrt{\frac{\sum (X_i - \bar{X})^2}{n}}$

41. Consider the following statements :

- I. A minimal sufficient statistic is always complete.
- II. A statistic that is independent of every ancillary statistic need not be complete.

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

42. Let  $X_i \sim \text{Binomial}(1, p)$ ;  $i = 1, 2, 3, \dots, n$ . For parameter  $p$ ,  $T = \sum_{i=1}^n X_i$  is :

- (a) Sufficient
- (b) Complete
- (c) Both sufficient and complete
- (d) Neither sufficient nor complete

43. Consider the following statements :

- I. If a statistic is sufficient for a class of distributions, then it is sufficient for any subclass of those distributions.
- II. A sufficient statistic is always a complete statistic.

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

44. Blackwellisation is a process of finding :

- (a) an unbiased estimator with higher consistency
- (b) a biased estimator with lesser variance
- (c) an unbiased estimator such that it is complete but not sufficient
- (d) an unbiased estimator with lesser variance

45. Let  $P_\theta[\theta \in S(X)] \geq 1 - \alpha$  for all  $\theta \in \Theta$  constitute a family of confidence sets. Which of the following statements is/are correct ?

- I.  $S(X)$  is a random set.
- II. For a given  $X = x$ , either  $S(X)$  covers  $\theta$  or it does not.

Select the answer using the code given below :

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

46. Let  $X$  follow Poisson distribution with parameter  $\lambda$ . Then, unbiased estimator of  $(-a)^x$  where 'a' is a constant, is :

- (a)  $e^{-\lambda}$
- (b)  $e^{-a\lambda}$
- (c)  $e^{-(a+1)\lambda}$
- (d)  $e^{-(a-1)\lambda}$

**Consider the following for the next two (02) items :**

Consider a random sample from a normal  $N(\mu, \sigma^2)$ .

47. What is the maximum likelihood estimator (MLE) of  $\hat{\mu}$  ?
- $\bar{x} / 2$
  - $\bar{x}$
  - $3\bar{x} / 2$
  - None of the above
48. What is the maximum likelihood estimator (MLE) of  $\sigma^2$  when  $\mu$  is not known ?
- $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$
  - $s^2 = \frac{1}{n+1} \sum_{i=1}^n (x_i - \bar{x})^2$
  - $s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$
  - None of the above

**Consider the following for the next two (02) items :**

Let the SPRT of strength  $(\alpha, \beta)$  and the boundary points (A, B) terminate with probability 1.

49. Which one of the following is correct ?

- $A \geq \frac{1-\beta}{\alpha}$
- $A \leq \frac{1-\beta}{\alpha}$
- $A \geq \frac{\alpha}{1-\beta}$
- $A \leq \frac{\alpha}{1-\beta}$

50. Which one of the following is correct ?

- $B \leq \frac{\beta}{1-\alpha}$
- $B \geq \frac{\beta}{1-\alpha}$
- $B \leq \frac{1-\alpha}{\beta}$
- $B \geq \frac{1-\alpha}{\beta}$

51. In the analysis of two-way classified data with m-observations per cell, where the factor A is at p levels and the factor B is at q levels, the degrees of freedom for the sum of squares due to error are :

- $pq(m-1)$
- $m(pq-1)$
- $m(p-1)(q-1)$
- $(m-1)(p-1)(q-1)$

52. A manufacturing company wishes to determine whether one of the three machines A, B and C is faster than the others in producing a certain output. The mean hourly output of the three machines is respectively 32, 37 and 27. It is given that at 5% level of significance, the critical difference between any two means to be significant is 5.62. Which one of the following is correct ?

- Machines A and B differ significantly
- Machines A and C differ significantly
- Machines B and C differ significantly
- No conclusion can be drawn

53. If the inner product of the column of the design matrix  $\mathbf{X}$  is equal to zero, then such experimental designs are said to be :

- Incomplete Block Design
- Non-orthogonal Design
- Orthogonal design
- Linear design

54. In the case RBD with 'b' blocks and 'v' treatments could be considered as a sample from a larger population, the appropriate two way model could be :

- (a) Two way fixed effect model of treatments and blocks
- (b) Two way random effect model of treatments and blocks
- (c) Two way mixed effect model of treatments as fixed effect and blocks as random effect
- (d) Two way mixed effect model of blocks as fixed effect and treatments as random effect

55. Consider the following statements :

- I. If  $A$  is an idempotent matrix, then  $\text{rank}(A) = \text{Trace}(A)$ .
- II. Given a matrix  $A$ , there exists a matrix  $A^+$ , with the property  $AA^+A = A$  and  $\text{trace}(A^+A) = \text{rank}(A)$ .

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

56. In the following Analysis of Variance Table, few entries are missing.

Source of variation	Degrees of freedom	Sum of squares	Variance ratio
Treatment	4	16.4	2.05
Variety	x	28	7
Error	y	z	
Total	15		

What are the values of x, y and z respectively ?

- (a) 4, 9, 18
- (b) 2, 9, 16
- (c) 9, 2, 18
- (d) 2, 9, 18

57. For a linear model,  $Y = X\beta + \epsilon$ ,  $E(\epsilon) = 0$ ,  $D(\epsilon) = \sigma^2 I$  where  $D$  stands for variance-covariance matrix and  $I$ , for the identity matrix of order n,

- I. The observational equation  $Y = X\beta$  is always consistent.
- II. The normal equation  $X'X\beta = X'Y$  always admits a solution.

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

58. For uncorrelated observations  $(y_1, y_2, \dots, y_n) = Y$ , we define a linear model-I.  $Y = X\beta + \varepsilon$  such that  $E(\varepsilon) = 0$ ,  $D(\varepsilon) = \sigma^2 I$ .  $E(Y) = X\beta$  and  $D(Y) = \sigma^2 I$ . We then define a more general model-II.  $E(Y) = X\beta$  and  $D(Y) = \sigma^2 G$ , where  $|G| \neq 0$  introduces correlations among the observations such that  $G$  is a known matrix,  $I$  is a unit matrix of order  $n$ ,  $D$  is a variance-covariance matrix and  $\beta, \sigma^2$  are unknown. Model-II can be reduced to Model-I by considering a transformation.

- (a)  $Z = G^{1/2} Y G^{-1/2}$
- (b)  $Z = G^{-1/2} Y$
- (c)  $Z = Y G^{-1/2}$
- (d) None of the above

59. Let the model be defined as

$$E(Y_1) = 2\beta_1 + \beta_2, E(Y_2) = \beta_1 - \beta_2,$$

$E(Y_3) = \beta_1 + \alpha\beta_2$ ; where  $Y_i$ 's are uncorrelated and have a constant variance  $\sigma^2$ . What is the value of  $\alpha$  so that the least square estimates of  $\beta_1$  and  $\beta_2$  are uncorrelated?

- (a) 2
- (b) 1
- (c) -1
- (d) -2

60. Consider the model  $Y_i = \alpha + (-1)^i \beta + u_i$  for all  $i = 1, 2, 3, \dots, 2n$ .  $u_i$ 's are independent random variables with  $E(u_i) = 0$  and  $V(u_i) = \sigma^2$  for all  $i$ . The least squares estimators of  $\alpha$  and  $\beta$  are given by :

$$I. \quad \hat{\alpha} = \frac{1}{2n} \sum_{i=1}^{2n} Y_i$$

$$II. \quad \hat{\beta} = \frac{1}{2n} \left[ \sum_{i=1}^n Y_{2i} - \sum_{i=1}^n Y_{2i-1} \right]$$

Which of the statements given above is/are correct?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

**Consider the following for the next two (02) items :**

Let  $X_i, Y_i$  and  $Z_i$ ;  $i = 1, 2, 3$  be 9 independent observations with common variance  $\sigma^2$ , and  $E(X_i) = \theta_1 - \theta_2, E(Y_i) = \theta_2, E(Z_i) = \theta_1 + \theta_2$ ;  $i = 1, 2, 3$ . Let  $X, Y$  and  $Z$  respectively denote  $\sum_{i=1}^3 X_i, \sum_{i=1}^3 Y_i$  and  $\sum_{i=1}^3 Z_i$ .

61. What is BLUE of  $\theta_1$  ?

- (a)  $\frac{1}{3}[X + Y]$
- (b)  $\frac{1}{9}[-X + Y + Z]$
- (c)  $\frac{1}{6}[X + Z]$
- (d)  $\frac{1}{9}[X - Y + Z]$

62. What is BLUE of  $\theta_2$  ?

- (a)  $\frac{1}{6}[-X + Z]$
- (b)  $\frac{1}{6}[X + Z]$
- (c)  $\frac{1}{9}[X - Y + Z]$
- (d)  $\frac{1}{9}[-X + Y + Z]$

*Consider the following for the next two (02) items :*

Let the regression model be given by  $Y_i = i\beta + u_i$ ;  $i = 1, 2, 3$  where  $u_i$ 's are independent random variables with  $E(u_i) = 0$  and  $V(u_i) = i\sigma^2$  for all  $i$ .

63. The best linear unbiased estimator of  $\beta$  for this model is given by :

- (a)  $\frac{1}{6}(Y_1 + Y_2 + Y_3)$
- (b)  $\frac{1}{14}(Y_1 + 2Y_2 + 3Y_3)$
- (c)  $\frac{1}{6}(Y_1 + 2Y_2 + 3Y_3)$
- (d)  $\frac{1}{36}(6Y_1 + 3Y_2 + 2Y_3)$

64. The variance-covariance matrix of the best linear unbiased estimator of  $\beta$  is given by :

- (a)  $\frac{\sigma^2}{14}$
- (b)  $\frac{\sigma^2}{6}$
- (c)  $\frac{49\sigma^2}{36}$
- (d)  $49\sigma^2$

65. Consider the model  $E(Y_{ij}) = \alpha_i + \beta_j$ ;  $i = 1, 2$  and  $j = 1, 2$ , where  $Y_{ij}$  are uncorrelated with common variance  $\sigma^2$ . Then the function  $l_1\alpha_1 + l_2\alpha_2 + m_1\beta_1 + m_2\beta_2$  is estimable if and only if :

- (a)  $l_1 - l_2 = m_1 - m_2$
- (b)  $l_1 + l_2 = m_1 + m_2$
- (c)  $l_1 + l_2 + m_1 + m_2 = 0$
- (d)  $l_1 + l_2 + m_1 + m_2 = 1$

66. Official Statistics are termed as "Public Good". Which of the following reasons justify this term ?

- I. They are financed from general tax revenue.
- II. Their use by one person does not affect the use by others.
- III. They are costly to produce but they are easily disseminated.
- IV. They are used only by public officials and private persons are denied access to them.

Select the answer using the code given below :

- (a) I, II, III and IV
- (b) I, II and III only
- (c) II and IV only
- (d) III and IV only

67. Consider the following :

- I. Power consumption
- II. E-way bills
- III. Rail freight traffic
- IV. Port cargo traffic

How many of the above are High Frequency Indicators ?

- (a) None
- (b) Only two
- (c) Only three
- (d) All four

68. Which one of the following is a correct definition of "Census House" in Indian Census ?

- (a) A house identified during the housing census which is used primarily for residential purpose.
- (b) A building used for residential or non-residential purpose which is marked and numbered for the population census.
- (c) A building or part of a building used or recognized as a separate unit because of having a separate main entrance which may be used for a residential or non-residential purpose or both.
- (d) A house in which a group of persons live together and take their meals from a common kitchen.

69. Consider the following statements about National Sample Surveys (NSS) :

- I. The 80<sup>th</sup> Round of NSS commenced from 1<sup>st</sup> January, 2025.
- II. The 80<sup>th</sup> Round of NSS covers a "Survey on Household Social Consumption-Health" and a "Comprehensive Modular Survey-Telecom".

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

70. Which of the following regarding the Indian Census Act, 1948 is correct ?

- I. The Population Census and the Housing Census both are taken under it.
- II. The Census may be taken in India or any part thereof.
- III. The Census may be taken whenever necessary or desirable.
- IV. The Census Act, 1948 was amended in 1994.

Select the correct answer using the code given below :

- (a) I, II and III only
- (b) II, III and IV only
- (c) I and IV only
- (d) I, II, III and IV

71. Which of the following constitute "normal" sources of financing the statistical infrastructure of a Nation ?

- I. Government budgets
- II. Multilateral trust funds
- III. Market borrowings
- IV. Bilateral donors and technical assistance

Select the correct answer using the code given below :

- (a) I, II and III
- (b) I, II and IV
- (c) I, III and IV
- (d) II, III and IV

72. In the Houselisting and Housing Census in India which precedes the population enumeration, which of the following information are canvassed ?

- I. Amenities available to the household
- II. Assets possessed by the household
- III. Details regarding floor, wall and roof of the Census house

Select the correct answer using the code given below :

- (a) I and II only
- (b) II and III only
- (c) I and III only
- (d) I, II and III

73. The Socio Economic Caste Census conducted in 2011 has been used primarily for which of the following purposes ?

- (a) Preparation of caste-wise list of households
- (b) Identification of beneficiaries for various welfare programmes of the Central Government
- (c) Selections of persons below poverty line in every State
- (d) Transfer subsidies by direct benefit transfer into bank account

74. Which of the following satisfy Time Reversal Test ?

- I. Marshall-Edgeworth Index
- II. Laspeyres Index
- III. Fisher's Ideal Index
- IV. Paasche Index

Select the correct answer using the code given below :

- (a) I, II and III
- (b) I and III only
- (c) I, III and IV
- (d) II and IV only

75. Which of the following are part of the UN Fundamental Principles of Official Statistics ?

- I. The laws, regulations and measures under which the statistical systems operate are to be made public.
- II. Bilateral and multilateral cooperation in statistics contributes to the improvement of systems of official statistics in all countries.
- III. Individual data collected by statistical agencies for statistical compilation, whether they refer to natural or legal persons, are to be strictly confidential and used exclusively for statistical purposes.

Select the correct answer using the code given below :

- (a) I and II only
- (b) II and III only
- (c) I and III only
- (d) I, II and III

76. Consider the following statements about United Nations Statistical Commission (UNSC) :

- I. It has 24 member countries elected by the United Nations Economic and Social Council.
- II. India is a member of UNSC at present and the four year term of India at UNSC began from January, 2024.

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

77. Which one of the following is **not** an objective of the Periodic Labour Force Survey (PLFS) conducted by National Statistical Office ?

- (a) To generate estimates of Labour Force Participation Rate
- (b) To generate estimates of Wage Rates and Average Earnings of Workers
- (c) To generate estimates of Worker Population Ratio
- (d) To generate estimates of Unemployment Rate

78. If consumption pattern of a society remains static for longer period, which method is most convenient and economic for compilation of index ?

- (a) Laspeyres' method
- (b) Paasche's method
- (c) Fisher's method
- (d) Dorbish and Bowley's method

79. The usual reference point of Population enumeration in India is normally :

- (a) 0-00 Hours of 1<sup>st</sup> January
- (b) 0-00 Hours of 1<sup>st</sup> March
- (c) 0-00 Hours of 31<sup>st</sup> March
- (d) 0-00 Hours of 31<sup>st</sup> December

80. Consider the following statements about Livestock Census in India :

- I. Livestock Census in India is conducted every 5 years.
- II. Livestock Census enumerates 25 species of wild and domesticated animals.

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

**SPACE FOR ROUGH WORK**

## SPACE FOR ROUGH WORK

**SPACE FOR ROUGH WORK**

1006721

1006757

SPACE FOR ROUGH WORK